

Mostly Sunny: A Forecast of Tomorrow's Power Index Research*

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Abstract: Power index research has been a very active field over the past few decades. Will this continue or have all important questions been solved? We argue that there are still many opportunities to conduct useful research with and on power indices. Positive and normative questions remain, calling for theoretical and empirical attention. Technical and technological improvements are likely to boost applicability.

Keywords: Power index analysis; economic perspectives and methodology; committee voting; optimal voting rules

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1. Introduction

The 750-page tome “Power, Voting, and Voting Power: 30 Years After” which was edited by Holler and Nurmi (2013) demonstrates that the past three decades of research on power indices have been very productive. Can this continue? Or as Manfred J. Holler put it when addressing a scientific community that has seen several (nominal) retirements of late: “Is there a future to power index research?”

The fact that two of the authors of this article have started to do research on power indices only in the 2010s supports our firm conviction that there exists a set of diverse topics on which progress can still be made, and will be made.

The two most prominent recent articles on allocating voting weights in two-tier systems (Barberá and Jackson, 2006; Koriyama et al., 2013) barely mention classical power measures. Social choice articles now appearing in top economics journals are concerned first and foremost with the welfare properties of voting systems; power comes as a distant second or even third (behind epistemic concerns). This might be regarded as a dark cloud in the sky of power index research. But welfarist approaches to voting, which focus on measures of success rather than pivotality, can be viewed as part of power index research defined in a sufficiently expansive way. The trend in economics journals therefore need not reflect negatively on their view of this research area. More generally, we see no evidence that voting power is facing greater suspicion from mainstream economists today than in the past.

We are convinced that prospects for power index research are no worse now than 30 years ago. Our academic weather forecast is therefore: mostly sunny! We expect power index research to have a productive future. The specific topics which we expect to be addressed can be grouped loosely into three areas. In Section 2, we focus on the positive analysis of voting bodies. We then adopt a more normative, design-oriented perspective in Section 3. A range of technical issues for which progress is likely are discussed in Section 4. We close with some concluding remarks in Section 5.

2. Positive Analysis

Voting is important for the quality of lives of billions of people. It shapes democratic participation at all levels of legislature and matters for decision making by boards or committees in the workplace. It also plays a role in non-governmental organizations, sports associations, and possibly even in the decision on the next family trip (e.g., Darmann et al., 2012).

As soon as voting and collective decision making involve some asymmetry – such as different weights or agenda setting rights – power indices turn out to be useful. They help to discover and quantify unevenness in the democratic playing field, which is easily obscured by vectors of weights, veto rules, thresholds, and quorums, and to assess the effects of possible rule changes.

Multinational organizations and other decision-making bodies which use weighted voting are evolving or being newly created (see, e.g., Belke and Styczynska, 2006, on the Governing Council of the European Central Bank). Modern communication technology facilitates the coordination of geographically dispersed actors in associations and interest groups which rely increasingly on formal decision rules rather than informal consensus. New proposals are being made to reform institutions to which power index analysis has long been applied, such as the Council of the EU, the UN Security Council, the Board of Governors of the IMF and the US Electoral College. Moreover, shareholder meetings of publicly traded companies remain arguably the most common but least studied kind of weighted voting bodies (see, e.g., Leech, 1988). So it is easy to affirm that the use of power indices in applied studies shall continue.

We predict that old distinctions and divisions in the literature will lose importance, however. For instance, there exists a wide spectrum between (a) pure *a priori* analysis, which purposely ignores any existing preference patterns in favor of the far-reaching independence and symmetry assumptions that underlie the Penrose-Banzhaf index (PBI; cf. Penrose, 1946, and Banzhaf, 1965) or Shapley-Shubik index

(SSI; cf. Shapley and Shubik, 1954),¹ and (b) more empirical *a posteriori* analysis which uses surveys and statistical techniques (e.g., the NOMINATE procedure developed by Poole and Rosenthal, 1985) in order to place specific voters, say, individual members of the US Congress or Supreme Court, on locations in a one- or multi-dimensional policy space in order to identify the critical Senators or judges for a given decision (see, e.g., Godfrey and Grofman, 2008, on health care reforms in the US). Many normative studies of two-tier voting systems take correlation between members of the same constituency behind the constitutional veil of ignorance. Why not do the same in positive analysis of, say, the IMF or EU?² The “veil of ignorance” is the most prominent motivation for independence and symmetry assumptions. But asymmetries among voters other than voting weights can be relevant. For instance, some EU members use proportional and others first-past-the-post systems in order to elect their governments and hence Council delegates; also election rules for members of the European Parliament still vary at the national level. Also the fact that some members of the IMF have preferential trade agreements or even share the same currency, while others do not, deserves to be accounted for. To some extent, power indices based on games with a priori unions or a restricted communication structure have always held a middle ground between pure *a priori* and *a posteriori* analysis (see Owen, 1977, and Myerson, 1977, for pioneering work), and we see scope for more work along these lines. We predict that increased public transparency and improved technology for analyzing voting data will focus research more towards the *a posteriori* end of the range.³

Other dichotomies will be fruitfully replaced by more pluralistic approaches too. Helpful as binary distinctions such as *a priori* and *a posteriori*, full approval vs. rejection, P-power vs. I-power, take-it-or-leave-it committees vs. bargaining committees, etc. may be, they can narrow one’s perspective. For example, while the

¹ See Felsenthal and Machover (1998) or Laruelle and Valenciano (2008a) for comprehensive overviews.

² Kaniowski (2008) has made promising progress in this direction.

³ See, for instance, the use by Badinger et al. (2014) of web scraping tools that are provided at <http://api.epdb.eu/> in order to gather a data set of almost 70,000 individual voting decisions of EU member states on more than 3,000 proposals.

attempt to delineate between the power to influence a collective decision (“I-power”) and the power to appropriate the surplus or “prize” generated by it (“P-power”) made by Felsenthal and Machover (1998) is certainly praiseworthy, the seemingly crisp conceptual juxtaposition blurs the fact that the two types of power are intertwined and the distinction is fuzzy at best. It can therefore be highly misleading to base a categorization of available power indices on it.⁴ It also makes a difference whether a decision making body can only adopt or reject an exogenous proposal (classified as a “take-it-or-leave-it committee” by Laruelle and Valenciano, 2008a) or if its members bargain in search of agreement over a set of feasible alternatives (a “bargaining committee” according to Laruelle and Valenciano). But it makes a similarly big difference whether the proposals before a take-it-or-leave-it committee are truly exogenous or proposed by a strategic agenda setter and likewise whether the alternatives negotiated in a bargaining committee are binary (e.g., declare independence or not), one-dimensional (e.g., tax rates, emission thresholds) or higher-dimensional (e.g., division of a monetary surplus).

With less “dichotomism” and a yet more diverse set of tools, future power index research will be better prepared to analyze the diverse voting bodies in the field. Ternary voting games that allow for abstention (Felsenthal and Machover, 1997) provide more accurate positive analysis of, say, power in the UN Security Council; quaternary dichotomous voting rules that allow for absences (with a quorum rule) provide yet more flexibility (Laruelle and Valenciano, 2012). Still more general frameworks for measuring power as pivotality or as outcome sensitivity have been developed by Bolger (1993) and Napel and Widgrén (2004).

The latter framework is suited also to analyzing collective decision-making in sequential legislative procedures, which may involve strategic interaction among

⁴ For instance, the PBI is commonly classified as a measure of I-power but also captures P-power in some situations (see Felsenthal and Machover, 1998, p. 45). The SSI is frequently classified as a measure of P-power but also captures I-power in relevant contexts (see Napel and Widgrén, 2008; Kurz et al. 2014a). In a wide range of circumstances PBI and SSI give very similar values; they always agree on the ranking of two players if these can be ordered by Isbell’s desirability relation.

the relevant players. The so-called “ordinary legislative procedure” of the European Union, formerly referred to as “codecision procedure”, has proposals made or amended by three different voting bodies in several readings and the possibility of bargaining in a “conciliation committee”. Positive analysis of the balance of power between European Commission, individual members of the Council, and the European Parliament therefore requires more than, say, a PBI calculation.⁵

The fact that conventional indices like the PBI or SSI are convenient to compute has probably biased applied research in their favor – to the detriment of more complicated but perhaps more appropriate methodology. This adverse fate has presumably also affected the nucleolus of voting games (see Peleg, 1968, and Schmeidler, 1969). Montero (2006) has provided a very convincing motivation for its use as a power measure when bargaining takes place in the shadow of a voting rule. To our knowledge, however, its application to the EU Council by Le Breton et al. (2012) has been the first and only. Fortunately, given that we expect progress on the computational ease of power index research (see Section 4), we predict a brighter future for both the nucleolus and analyses of sequential voting procedures.

The blunt question “Which is the right power index?” has fortunately been replaced by more subtle ones, asking which of various properties that go with distinct indices or methods fit a specific application best. Distinct members of the community naturally differ in their answers. The Holler-Packel index (see Holler and Packel, 1983), for instance, is vigorously advocated by some while others group it under “minor indices” (Felsenthal and Machover, 2005; 1998, p. 245) and hold that “any reasonable measure of a priori voting power ... must respect dominance” (which the Holler-Packel index does not). While many scholars have expressed a pronounced preference for the PBI over the SSI at workshops and conferences others have expressed the opposite preference.

⁵ See Mayer et al. (2013) on analysis of the codecision procedure for EU28, and Felsenthal et al. (2003, p. 490) on the “informational poverty” of traditional power indices.

This subjectivity and apparent arbitrariness is a cloud in the sky of power index research, at least from the perspective of many outsiders. Fortunately, the literature has started to address the details of what constitutes power in which types of voting situations and what is the predictive value of power indices on a wider empirical basis. So far, laboratory experiments have been the method of choice. They provide maximal control over the aspects of a voting situation that determine a power index's potential value added. Montero et al. (2008), for instance, report on an experiment that empirically demonstrates the paradox of new members, which was a key prediction of power index analysis. Geller et al. (2004) provide evidence that the SSI and the PBI describe power well in divide-the-dollar weighted voting games. More experimental power index research can be expected – someday perhaps even in the field – and will be most welcome.

A related area in which future empirical research could be promising pertains to preferences for different voting systems. Can such preferences be explained by the respective distribution of voting power, as measured by a particular index? How do people trade off procedural concerns (e.g., for equal swing probabilities) and personal success propensities? Weber (2014) provides first evidence that subjects have a preference for voting systems that allocate Shapley-Shubik power to group representatives proportionally to group size. These systems are preferred over ones more in line with Penrose's square root rule to an extent that is not explicable by classic consequentialism.

3. Normative Analysis

The increased pluralism which we predict for positive analysis has its natural analogues – and in some cases: precedents – in normative analysis. We already pointed to an improved account of pre-existing asymmetries in constitutional analysis. If, for instance, it is a restriction for the design of a two-tier voting system that the existing population partition cannot be changed into constituencies of equal sizes, this usually has a reason. Often the constituents have distinct regional identities, languages, religious denominations, or ethnicities and hence differ more

from each other between than within constituencies. It is then appropriate to take positive correlation within constituencies behind the veil of ignorance rather than presume independence. More generally, power index research will do well to go beyond the assumption of unfettered symmetry.

Investigations of the “optimal” design of two-tier voting systems have branched into numerous different objective functions since the seminal investigation by Penrose (1946). Equality of voting power or of expected utility across individuals, maximal welfare under different utilitarian assumptions, minimal majority deficit or discrepancy between the outcomes of a two-tier vs. a direct voting system⁶, and minimal distance between weights and induced voting powers have all been considered.⁷ In spite of this diversity in objectives, the great majority of the studies have remained faithful to Penrose’s original binary setup, which considered a collective decision between two exogenously given alternatives (a random legislative proposal vs. the status quo). Neither voter abstention nor the possibility of three or more ordered policy alternatives is considered. Also the case of proposals arising endogenously from strategic agenda setting or from two-party competition remains to be explored.

We forecast more departures from the conventional binary focus than the few that have been undertaken. These include Laruelle and Valenciano (2008b) and Le Breton et al. (2012), who have analyzed delegated bargaining over a simplex of policy alternatives, i.e., problems of rent division. Maaser and Napel (2007; 2012; 2014) have used Monte Carlo simulation in order to study influence-based, majoritarian, and welfarist objective functions in a median voter environment with an interval of policy options. Asymptotically optimal assignments of weights in the latter environment have been analytically characterized by Kurz et al. (2014a) for

⁶ The “discrepancy” may be operationalized, for instance, by the probability of obtaining different outcomes or the average distance between direct and indirect voting outcomes.

⁷ This list should grow further. Design of two-tier voting systems with epistemic goals or explicit minority protection constraints are promising research areas. It is also an open issue how to simultaneously cope with multiple normative criteria. For instance, equitable representation in UNO or IMF can relate to countries’ population sizes but also to financial and other contributions to the common objective. No single “optimal rule” may exist; but this raises the question of which rules are Pareto-optimal with respect to any given set of criteria.

a democratic fairness objective similar to Penrose's. Because differences in population numbers have quite different statistical effects in case of only two policy options vs. a larger finite number vs. an interval, it is surprising that the pattern obtained from binary setups has re-appeared for a continuum of alternatives. Namely, optimal weights relate to the square root of population sizes in case of independent voters but simple proportionality is called for once opinions of constituency members are at least mildly correlated. The intermediate case with a finite number of alternatives greater than two has not been systematically studied so far. Our preliminary computations indicate that the square root rule for independent and identically distributed (i.i.d.) voter attitudes may actually break down. Future research will clarify whether famous square root results are knife-edge not only with respect to the i.i.d. assumption but perhaps also with regard to allowing only two policy options.

A one-dimensional interval of alternatives is sufficient to conduct an analysis of economic questions that would otherwise not be covered (e.g., scope of regulation, spending on climate change mitigation, monetary policy); but it would be desirable to extend the analysis to multi-dimensional spaces. One possibility to deal with two or more dimensions could be to use point solutions, like the Copeland winner (also known as strong point in spatial voting analysis), which exist even if the generalized median voter does not. Another possibility is to assume an exogenous ordering of dimensions on which individuals vote sequentially (see De Donder et al., 2012).

So far, power index research and its normative applications to representative democracy have by-and-large stayed closely in the tracks of winner-takes-all systems, which are easily modeled by weighted voting games. Other electoral systems like proportional representation or mixed-member systems have been neglected. We forecast that this will change. Edelman (2004), for instance, has considered the ideal composition of a legislature that contains representatives from equipopulous districts and some number of at-large representatives if the objective is to maximize the total Banzhaf power of individual citizens. Other scenarios with two (or even more) types of legislators, representing different

interests of the electorate, are conceivable and will be studied in the future. What, for instance, would a mixed-member legislature or a two-chamber legislature ideally look like if voters have interests along regional and federal dimensions, which can be either independent or aligned in complicated ways?

4. Tools and Technical Issues

As in research more generally, the types of power investigations carried out depend on the available mathematical and computational tools. Substantial progress has been made regarding the efficient computation of power indices. Free software packages make it easy for applied researchers to calculate power indices without writing their own programs or to adapt published code to a specific application (see, e.g., Macé and Treibich , 2012).

Understandably, the availability of software is biased towards the most popular conventional indices, namely the PBI and the SSI. But popularity is also a consequence of availability. We are unaware, for example, of any online tool which allows an applied researcher to compute the nucleolus. The 27-member assembly considered by Le Breton et al. (2012) presents an almost insurmountable computational obstacle for non-experts. So we see a future for more easy-to-use software, especially for the computation of technically more demanding constructs.⁸ Algorithms for power analysis based on convex policy spaces are still in their infancy.

There is room for improvements even in the computation of SSI and PBI. Namely, the efficiency of the most widely used generating function approach (see Alonso-Meijide et al., 2012) relies heavily on working with small integer weights. This is in stark contrast with population figures in the millions being used as weights in the EU Council. Large weights can also arise when trying to implement Penrose's

⁸ We would also include in this category the power indices designed for voting games with a priori unions, with restricted communication, and with hierarchies (see, e.g., van den Brink and Steffen, 2012) as well as the minimum sum representation index which was recently proposed by Freixas and Kaniowski (2014).

square root rule as well as possible. Techniques have recently been developed to compute equivalent representations with smaller or even the minimum integer weights (see, e.g., Kurz, 2012a). These may in the future prove worthwhile for index computations, too. And they may be extended to two- or three-dimensional voting weights, as they are currently applied by the EU Council.

Another important technical issue is the so-called “inverse problem” of finding a voting rule that induces a given distribution of power according to some power index as closely as possible for a given notion of distance. If one does not want to rely on simple heuristics (for which it is mostly impossible to prove certain desirable qualities such as a known maximal distance to the optimal solution), the problem is computationally very expensive (see De et al., 2012, and Kurz, 2012b). Progress can still be made regarding a better understanding of common heuristics (Kurz and Napel, 2014) and regarding the efficient – ideally also user-friendly – implementation of exact algorithms. The usefulness of, e.g., the integer linear programming techniques employed by Kurz (2012b) will benefit from steadily improving computer hardware; it is also conceivable that the complete list of distinct weighted voting games with up to nine players will in coming years become searchable online.

We also forecast progress in the pure theory of power indices. The distribution of inducible power vectors within the unit simplex remains something of a mystery even for the classical PBI or SSI. Alon and Edelman (2010) have recently shown that even for large numbers of players some target PBI distributions can be reached only with a large and constant relative error. Their path-breaking work is in the process of being extended to other power indices (see Kurz, 2014).

Another theoretical issue of practical relevance is the possible coincidence of voting weights and power – either in an exact or asymptotic sense. It was shown only recently that the nucleolus of non-oceanic weighted majority games converges to the relative weight distribution (see Kurz et al., 2014b). The same article provides a new sufficient condition for exact coincidence of nucleolus and weights, which future research can presumably weaken. Coincidence of power and

weights has also been studied recently by Houy and Zwicker (2014) for the PBI. Analogous findings for the SSI remain to be developed. The first attempt by Leech (2013) to develop a comprehensive asymptotic result for power indices, which would cover both oceanic and non-oceanic games, inadvertently misstates rather than generalizes the findings by Lindner and Machover (2004). But the goal is worthwhile, and we forecast that it will be achieved in future research.

5. Concluding Remarks

Our own research interests have certainly biased the identification of topics for which we expect fruitful power index research. That the collection is obviously too big an agenda for us alone, however, indicates the wide scope for continuing with or moving into power index research.

This scope becomes even wider if one also considers topics that are more distantly related to voting power. For instance, the quantifications of causal responsibility by Braham and Holler (2009), Braham and van Hees (2009) or Felsenthal and Machover (2009) draw more or less explicitly on power analysis of non-strategic binary voting. Carrying methods and insights from non-binary strategic voting over into this domain looks promising. The domain of conventional power index research has also been left by Koster et al.'s (2014) investigation of the predictive value of knowing an individual voter's decision or voting inclination. Taking the latter as input into a model of an opinion formation process could merge traditional power analysis with the analysis of social dynamics and networks.

Finally, indices and techniques that have been popularized by voting applications can prove useful in completely unrelated contexts. For example, Kovacic and Zoli (2013) compute the PBI with relative population shares of different ethnicities as "weights" in an analysis of ethnic conflict. They find that a PBI-based approach can explain the onset of conflict better than existing indices of ethnic diversity.

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